MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

EMT1016 – ENGINEERING MATHEMATICS I

(All Sections / Groups)

15 OCTOBER 2018
9AM – 11AM
(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **3 pages** (including cover page) with **4** Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

(a) Evaluate the following limits, if they exists.

(i) $\lim_{x\to 2}\dfrac{x-2}{\sqrt{x+2}-2}$. [4 marks]

(ii) $\lim_{x\to 1} x^{1/(x-1)}$. [4 marks]

(b) If $x+\sqrt{y}=\left(x^2y+4\right)^3$, use implicit differentiation to find $\dfrac{dy}{dx}$. [4 marks]

(c) Evaluate the integral $\int\dfrac{3x+1}{(x+3)(x-1)}dx$. [6 marks]

(d) Sketch the graphs of $y=|x|$ and $y=-x^2+2$. Hence, find the area of the region bounded by the two curves. [7 marks]

Question 2

(a) Let $f(x,y)=\dfrac{x^2y}{2x^4+y^2}$:

(i) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the x-axis. [2 marks]

(ii) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the line $y=x^2$. [2 marks]

(iii) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Justify your answer. [2 marks]

(b) A hardware store sells two brands of paint, brands A and B. An analysis of the sales figures indicates that the total demand Q (litres per month) is modeled by:

$$Q(x,y)=210-12x^2+18y,$$

where x and y are the prices of brand A and brand B per litre, respectively. A separate analysis indicates that t months from now, brand A will cost $x=25+0.18t$ (RM/litre) and brand B will cost $y=35+0.3\sqrt{t}$ (RM/litre). Use Chain Rule to find $\dfrac{dQ}{dt}$ at $t=9$, which is the rate of change of total demand 9 months from now. [7 marks]

(c) Let $f(x,y,z)=x+y+\left(\sqrt{2}\right)z$. Using the method of Lagrange multiplier, determine the minimum and maximum values of $f(x,y,z)$ on the surface of a sphere $x^2+y^2+z^2=4$. [12 marks]

Continued...

Question 3

(a) The impedance of two parallel AC circuits is given by:

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}, \text{ where } Z_1 = 5 - i \text{ and } Z_2 = 3 + i.$$

(i) Express Z_T in Cartesian form. [4 marks]

(ii) Express Z_T in exponential form. [3 marks]

(b) Find all three distinct roots for $w = \sqrt[3]{1-i}$. [6 marks]

(c) The Maclaurin series for $\cos x$ is

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad x \in \Re.$$

Find the Maclaurin series for $f(x) = x\cos(x^2)$, then provide the first four terms of this series. [6 marks]

(d) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n-1)!}{n+1}(x-2)^n$.

[6 marks]

Question 4

A periodic function $f(x)$ of **period 2** is defined over $[-1,1)$ by

$$f(x) = \begin{cases} 0, & -1 \le x < 0, \\ 3x, & 0 \le x < 1. \end{cases}$$

(a) Sketch the graph of $f(x)$ from $x = -4$ to $x = 3$. [4 marks]

(b) Compute the Fourier coefficients (i.e. a_0, a_n and b_n) of $f(x)$. Show that the Fourier series representing $f(x)$ is given by

$$\frac{3}{4} - \frac{6}{\pi^2}\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}\cos(2k-1)\pi x + \frac{3}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\sin n\pi x.$$

[17 marks]

(c) To what value will the Fourier series for $f(x)$ converge at the point $x = 1$? Hence, show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots.$$ [4 marks]

End of paper.